2021 ECNU Campus Invitational Contest

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Acknowledgement

Developers

- Bochao Mao
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Testers

- Weiyao Huang
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- Hanshuo Zhang
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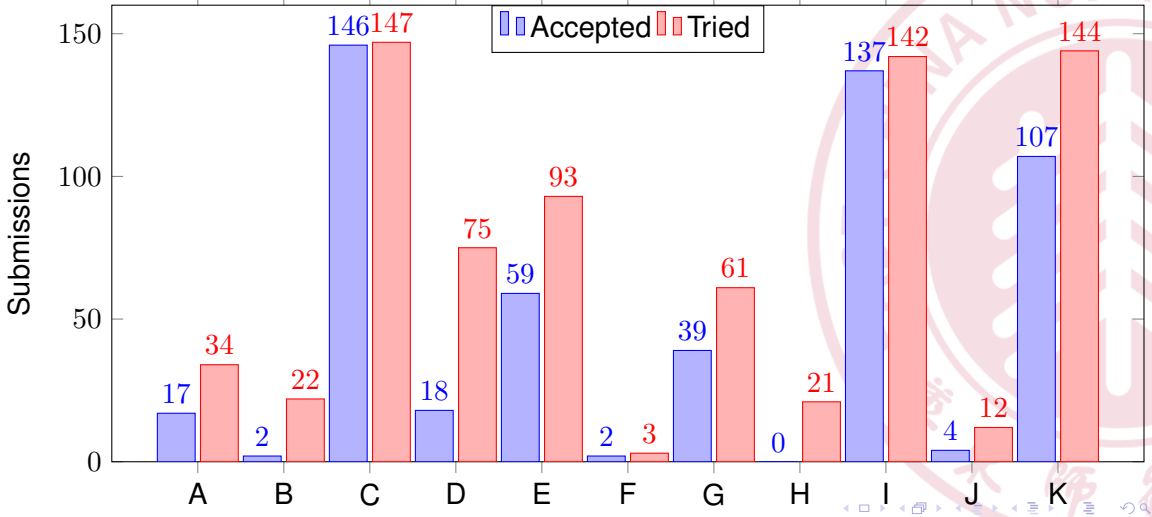
Judges' Anticipation

- Very easy: C
- Easy: E, I, K
- Medium easy: A, D, G, J
- Medium hard: B, F
- Hard: H

Summary (Onsite)



Congratulate Top-10 Competitor

#		=	罚时	A	B	C	D	E	F	G	H	I	J	K
1	519-48 10195102457 祝润天 395#0105	8	676	+ 74		+ 7	+2 141	+ 25		+ 85		+ 14	+2 238	+ 10
2	511-40 10185101248 邵煜 395#0040	8	846	+4 171	+6 207	+1 9	+ 127	+ 28		+ 35		+ 14	-3	+1 11
3	511-35 10185101232 王朝扬 395#0035	8	1273	+6 235		+ 9	+1 124	+1 80	+20 196	+ 36		+ 7		+ 22
4	511-49 10185101281 包梁 395#0049	7	426	+3 182		+ 8	+ 71	+ 23		+ 43		+ 12		+1 4
5	527-35 10205102432 朱睿诚 395#0149	7	514	+2 170		+ 7	+ 105	+ 56		+ 41		+1 19		+2 14
6	519-22 10195101452 章兆萌 395#0079	7	604	+4 159		+ 3	+2 185	+ 28		+1 55		+ 19		+ 12
7	527-57 10175102210 李思 395#0171	7	628	+ 136		+ 9	+2 187	+ 46		+1 145		+ 24	-2	+ 17
8	511-56 10185102153 汪子凡 395#0056	7	668	+1 183		+1 13	+3 227	+ 42		+1 53		+ 18		+ 9
9	519-2 10185102223 汪杰 395#0059	7	672	+ 130		+1 21		+ 25		+ 37		+ 15	+8 223	+1 18
10	511-6 10175102209 吕熠强 395#0006	7	677	+2 206		+ 42	+3 154	+1 62		+1 17		+ 23		+ 30

A. Abstract Algebra

Tags

Linear algebra, Constructive algorithm

Solutions

- If $c = 0$, we have $a = d = 1$ or $a = d = -1$.
- $\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = A^b$, $\begin{bmatrix} -1 & b \\ 0 & -1 \end{bmatrix} = B^2 A^{-b}$

A. Abstract Algebra (Cont.)

Solutions

- $a = 0$ then $\begin{bmatrix} 0 & b \\ c & d \end{bmatrix} = B^{-1} \begin{bmatrix} c & d \\ 0 & -b \end{bmatrix}$
- $b = 0$ then $\begin{bmatrix} a & 0 \\ c & d \end{bmatrix} = B^{-1} \begin{bmatrix} -d & c \\ 0 & -a \end{bmatrix} B^{-1}$
- $d = 0$ then $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix} = \begin{bmatrix} -b & a \\ 0 & c \end{bmatrix} B^{-1}$

First Solved: Runtian Zhu, 1:14(+)

B. Bracelet

Tags

Brute force, Strings

Solutions

- Only three cases:
  - $i = m$
  - i is a substring of n
  - The suffix of i and the prefix of $i + 1$ form n
- We can enumerate directly due to n have at most 18 numbers

First Solved: Yunfan Li, 1:40(+5)

C. Countdown

Tags

Date

Solutions

- 189
- Using excel, python or fingers can quickly get the answer

First Solved: Xiaobo Guo, 0:03(+)

D. Divide

Tags

Math

Solutions

- $a \cdot (a+1) \cdots b \mid c \cdot (c+1) \cdots d \Leftrightarrow (c-1)! \cdot b! \mid (a-1)!d!$
- Define $F_p(x) = y$ for prime p that $p^y \mid x$ but $p^{y+1} \nmid x$
- $a \mid b$ iff. $\forall p, F_p(a) \leq F_p(b)$ which p is a prime
- We can first get the prime within 10^7, then calculate the $F_p(x!)$ to get the answer

First Solved: Liang Bao, 1:11(+)

E. Edge Game

Tags

Data structure, DFS

Solutions

- Win or lose only depends on the parity of the distance between the two nodes.
- Use LCA to get the distance of the path.
- The easier solution is coloring the nodes to make the adjacent nodes have different colors, then judge parity of the distance through the color of two nodes.

First Solved: Yunfan Li, 0:17(+)

F. Function-Cuber

Tags

Math, Interactive

Solutions

- Ask a query $(x, x+1)$, we will get the value of $s + a_{x-1} - a_x + a_{x+1} - a_{x+2} - 1$. Thus we get the value of $a_{x-1} - a_x + a_{x+1} - a_{x+2}$
- Ask a query $(x, x+2)$, we will get the value of $s + a_{x-1} - a_{x+3}$. Thus we get the value of $a_{x-1} - a_{x+3}$

F. Function-Cuber (Cont.)

Solutions

- Without loss of generality, lets say $a_0 = a_{n+1} = 0$
- Ask queries like $(x, x+2)$ where $x = 1, 5, 9, \cdots = 4k+1$. Since we know a_0, we can get the value of a_4. Then a_8, a_{12}, $\cdots$ a_{4k} also can be determined
- As we also know a_1, so use similar method we determined the value of all a_{4k+1}
- After that, we can notice for every 4 consistent elements $a_x, a_{x+1}, a_{x+2}, a_{x+3} (1 \le x \le n-3)$, there are at least 2 elements which has been determined. So we ask $(x+1, x+2)$ to get the sum or difference of the rest unknown elements(that is a **equation**). Ask such queries several times to do **elimination** until we use a single unknown quantity to represent all other unknown number

F. Function-Cuber (Cont.)

Solutions

- Finally, we set up the equation $f(a) = \sum_{i=1}^{n-1} a_i a_{i+1} = s$ to get the value of the only unknown quantity (may use the formula of quadratic equation)
- Time complexity is $O(n)$. We can use less than $n + 5$ queries

First Solved: Chaoyang Wang, 3:16(+20)

G. Group QQ Speed

Tags

Constructive algorithm, Math

Solutions

- If everyone is in the same group, we have to have $n + 1$ maps due to everyone can ban different maps
- Otherwise, we need at most 3 maps
- But if only one person in each group, we just need 2 maps

First Solved: Yiqiang Lv, 0:17(+1)

H. Histogram in 3D

Tags

Data structure, Divide and conquer, Convex hull, Two pointers

Solutions

- Let $x(i,j) = \min(x_i, \cdots, x_j)$ and $y(i,j) = \min(y_i, \cdots, y_j)$
- We want to find the (i,j) which maximises $x(i,j) \cdot y(i,j) \cdot (j-i+1)$
- We use divide-and-conquer approach to solve this case

H. Histogram in 3D (Cont.)

Solutions

- For each turn, we need calculate the interval (i, j) that $i \in [l, mid]$ and $r \in (mid, r]$
- We should consider four cases:
  1. $x(l, r) = x(l, mid)$ and $y(l, r) = y(l, mid)$
  2. $x(l, r) = x(l, mid)$ and $y(l, r) = y(mid + 1, r)$
  3. $x(l, r) = x(mid + 1, r)$ and $y(l, r) = y(l, mid)$
  4. $x(l, r) = x(mid + 1, r)$ and $y(l, r) = y(mid + 1, r)$

H. Histogram in 3D (Cont.)

Solutions

- First and forth cases are simple
- In first case, $x(l,r) = x(l,mid)$ and $y(l,r) = y(l,mid) \Rightarrow x(mid+1,r) \geq x(l,mid)$ and $y(mid+1,r) \geq y(l,mid)$
- Then we can use the two pointers method to solve
- Forth case is similar to the first case

H. Histogram in 3D (Cont.)

Solutions

- In second case, $x(l,r) = x(l,mid)$ and $y(l,r) = y(mid+1,r) \Rightarrow x(mid+1,r) \geq x(l,mid)$ and $y(mid+1,r) \leq y(l,mid)$
- We have $x(l,mid) \cdot y(mid+1,r) \cdot (r-l+1) = (x(l,mid) \cdot (-l+1), x(l,mid)) \cdot (y(mid+1,r), y(mid+1,r) \cdot r)$
- $(x(l,mid) \cdot (-l+1), x(l,mid))$ only depends on l, and $(y(mid+1,r), y(mid+1,r) \cdot r)$ only depends on r

H. Histogram in 3D (Cont.)

Solutions

- For a fixed l, we want to find r that maximises the dot product
- We can see the point $(x(l, mid) \cdot (-l + 1), x(l, mid))$ move counterclockwise as l increases, so the optimal point on the hull will also move counterclockwise
- We can sort the point by the first coordinate and build a segment tree that has the convex hull of the corresponding points in each node
- Then we can keep the last query's optimal point to make the answering in linear complexity

First Solved: N/A

I. I Love You

Tags

Strings

Solutions

- If we can change s to t by removing some substrings,
  it means t is a subsequence of t

First Solved: Siyang Weng, 0:03(+)

J. Just the Chosen One

Tags

Probability

Solutions

- When $k \geq m$, the answer is $(\frac{1}{k} + \frac{1}{k+1} + \ldots + \frac{1}{n}) \cdot m$
- When $k < m$, the answer is $(m - k) + (\frac{1}{m} + \frac{1}{m+1} + \ldots + \frac{1}{n}) \cdot m$
- Since n can be very large, you can use $\sum_{i=1}^{n} \frac{1}{i} \sim \ln n$ to estimate the value of $\sum \frac{1}{i}$. More specifically, you can directly calculate $\sum_{i=1}^{n} \frac{1}{i}$ if $n \leq 10^7$, and if $n \geq 10^7$, you can assume that the answer is $\ln n - \ln 10^7 + \sum_{i=1}^{10^7} \frac{1}{i}$

First Solved: Yuanqing Chen, 03:37(+5)

K. K-Primes

Tags

Math

Solutions

- Every even number is not a prime, except 2
- If $l \neq 2$ then $[l, l + 2k)$ have at most k primes
- If $l = 2$ then $[l, l + 2k)$ have at most k primes except $\{2, 3\}$, $\{2, 3, 4, 5\}$ and $\{2, 3, 4, 5, 6, 7\}$

First Solved: Chenkai Wang, 0:03(+)

End

Thanks for attention!